

A TWO-FORM FORMULATION OF THE VECTOR-TENSOR MULTIPLY IN CENTRAL CHARGE SUPERSPACE

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ABSTRACT. A two-form formulation for the $N = 2$ vector-tensor multiplet is constructed using superfield methods in central charge superspace. The $N = 2$ non-Abelian standard supergauge multiplet in central charge superspace is also discussed, as is with the associated Chern-Simons form. We give the constraints, solve the Bianchi identities and present the action for a theory of the vector-tensor multiplet coupled to the non-Abelian supergauge multiplet via the Chern-Simons form.

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1. INTRODUCTION

In a recent paper [1], we constructed the $N = 2$ vector-tensor supermultiplet [2, 3] as a constrained supergauge multiplet using superfield techniques in $N = 2$ central charge superspace [1, 4, 5]. In that work, the gauge component field of this supermultiplet was introduced via an Abelian gauge connection superfield and the antisymmetric tensor component field derived as a consequence of a particular choice of constraints. In [1], we presented the theory for a free, non-interacting vector-tensor multiplet, preferring to concentrate on the important theoretical issues, such as the choice of constraints, the central charge hierarchy and construction of a supersymmetric action. Our work, however, was motivated by recent results in superstring theory [6], where the interacting vector-tensor multiplet emerged as

being of fundamental importance. The coupling of the vector-tensor multiplet to supergravity and to vector multiplets has also recently been considered [7, 8]. It was shown that this requires the gauging of central charge, leading to a Chern-Simons coupling between the vector-tensor multiplet and the $N = 2$ standard vector multiplet. With this in mind, it is the purpose of this paper to present the general theory for the coupling of the $N = 2$ vector-tensor supermultiplet to both Abelian and non-Abelian $N = 2$ standard supergauge multiplets via the associated super Chern-Simons forms. To do this, it is convenient to present an alternative construction for the vector-tensor multiplet as a constrained super two-form. We do this using superfield techniques in $N = 2$ central charge superspace. In this approach, the two-form component field of this supermultiplet is introduced via a two-form superfield and the gauge component field derived as a consequence of a particular choice of constraints. We will also present, in central charge superspace, the general theory for the $N = 2$ non-Abelian standard supergauge multiplet and its associated Chern-Simons form. Finally, we can combine these two formalisms to construct the general central charge superspace theory for the coupling of the vector-tensor supermultiplet to an Abelian or non-Abelian standard supergauge multiplet via the Chern-Simons form. We explicitly give the constraints, solve the Bianchi identities and give the superfield action for this coupled theory.

2. $N = 2$ CENTRAL CHARGE SUPERSPACE

As discussed in [9, 10, 1], $N = 2$ central charge superspace is a space with coordinates $z^M = (x^m, \theta_i^\alpha, \bar{\theta}_{\dot{\alpha}}^i, z, \bar{z})$ where x^m , z , and $\bar{z} = z^*$ are commuting bosonic coordinates while θ_i^α and $\bar{\theta}_{\dot{\alpha}}^i = (\theta_{\alpha i})^*$ are anticommuting fermionic coordinates. Taylor-series expansion of a generic superfield in the θ coordinates terminates after a finite number of terms due to the anticommuting nature of θ . On the other hand, the expansion in z and \bar{z} never ends. This means there are, a priori, an infinite number of component fields in a general superfield. These can, however, be reduced to a finite number off-shell by applying appropriate constraints.

Translations in this superspace are generated by the supercovariant differential operators ∂_a , ∂_z , $\partial_{\bar{z}}$, and

$$\begin{aligned} D_\alpha^i &= \frac{\partial}{\partial \theta_i^\alpha} + i\sigma^a_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}i} \partial_a - i\theta_\alpha^i \partial_z, \\ \bar{D}_{\dot{\alpha}i} &= -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}i}} - i\theta_i^\alpha \sigma^a_{\alpha\dot{\alpha}} \partial_a - i\bar{\theta}_{\dot{\alpha}i} \partial_{\bar{z}}, \end{aligned} \tag{2.1}$$

where $\bar{D}_{\dot{\alpha}i} = -(D_{\alpha}^i)^{\dagger}$. The anticommutation relations for these operators are

$$\begin{aligned}\{D_{\alpha}^i, \bar{D}_{\dot{\alpha}j}\} &= -2i\delta_j^i \sigma^m_{\alpha\dot{\alpha}} \partial_m, \\ \{D_{\alpha}^i, D_{\beta}^j\} &= -2i\epsilon_{\alpha\beta} \epsilon^{ij} \partial_z, \\ \{\bar{D}_{\dot{\alpha}i}, \bar{D}_{\dot{\beta}j}\} &= 2i\epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{ij} \partial_{\bar{z}}.\end{aligned}\tag{2.2}$$

The supervielbein e_A^M of this superspace is defined as the matrix that relates the supercovariant derivatives $D_A = (\partial_a, D_{\alpha}^i, \bar{D}_{\dot{\alpha}i}, \partial_z, \partial_{\bar{z}})$ and the ordinary partial derivatives

$$D_A = e_A^M \frac{\partial}{\partial z^M}.\tag{2.3}$$

The torsion T^A is defined as

$$T^A = de^A = \frac{1}{2}e^C e^B T_{BC}^A.\tag{2.4}$$

The non-vanishing components of the torsion are found to be

$$\begin{aligned}T_{\alpha\dot{\alpha}}^{ij a} &= T_{\dot{\alpha}\alpha}^{ji a} = -2i\epsilon^{ij} \sigma^a_{\alpha\dot{\alpha}}, \\ T_{\alpha\beta}^{ij z} &= T_{\beta\alpha}^{ji z} = 2i\epsilon^{ij} \epsilon_{\alpha\beta}, \\ T_{\dot{\alpha}\dot{\beta}}^{ij \bar{z}} &= T_{\dot{\beta}\dot{\alpha}}^{ji \bar{z}} = 2i\epsilon^{ij} \epsilon_{\dot{\alpha}\dot{\beta}}.\end{aligned}\tag{2.5}$$

3. VECTOR-TENSOR MULTIPLY: GAUGE THEORY FORMULATION

In this section we briefly describe the super-gauge theory formulation of the vector-tensor multiplet presented in [1]. We begin by considering the geometrical form of super-gauge theory in superspace with central charge. We then find a suitable set of constraints on the super field strengths to reproduce the field content of the vector-tensor multiplet. Here we restrict our attention to Abelian gauge theories, returning to a discussion of non-Abelian theories below. Let us introduce an hermitian connection $A = dz^M A_M = e^A A_A$. The hermiticity of the connection implies

$$A = dx^a A_a + d\theta_i^{\alpha} A_{\alpha}^i + d\bar{\theta}_{\dot{\alpha}}^i \bar{A}_{\dot{\alpha}}^i + dz A_z + d\bar{z} A_{\bar{z}},\tag{3.1}$$

where A_a is real, $\bar{A}_{\dot{\alpha}}^i = (A_{\alpha}^i)^{\dagger}$, and $A_{\bar{z}} = A_z^{\dagger}$. The curvature two-form is defined as

$$F = dA = \frac{1}{2}e^B e^A F_{AB}.\tag{3.2}$$

The curvature tensor F is subject to the Bianchi identities $dF = 0$. It is natural to adopt a set of constraints which set the pure spinorial part of the curvature tensor to zero. That is

$$F_{\alpha\beta}^{ij} = F_{\alpha\dot{\beta}}^{ij} = F_{\dot{\alpha}\dot{\beta}}^{ij} = 0.\tag{3.3}$$

We would like to explore the consequences of (3.3). To do so we must solve the Bianchi identities subject to these constraints. The result is that all the components of the curvature tensor F_{AB} are determined in terms of a single superfield $F_{\alpha\bar{z}}^i$ and

its hermitian conjugate $F_{\dot{\alpha}i\bar{z}} = (F_{\alpha\bar{z}}^i)^\dagger$. Henceforth, we denote these superfields by W_α^i and $\bar{W}_{\dot{\alpha}i}$. In particular $F_{\alpha z}^i = 0$ and

$$F_{ab} = -\frac{1}{16}i\epsilon_{ij}\bar{\sigma}_a^{\dot{\alpha}\alpha}\bar{\sigma}_b^{\dot{\beta}\beta}\left(\epsilon_{\dot{\alpha}\dot{\beta}}D_\beta^jW_\alpha^i + \epsilon_{\alpha\beta}\bar{D}_{\dot{\beta}}^j\bar{W}_{\dot{\alpha}}^i\right). \quad (3.4)$$

$$F_{\dot{\alpha}a}^i = -\frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}_a^{\dot{\beta}\alpha}W_\alpha^i, \quad (3.5)$$

$$F_{a\bar{z}} = -\frac{1}{8}i\epsilon_{ij}\bar{\sigma}_a^{\dot{\beta}\alpha}\bar{D}_{\dot{\beta}}^jW_\alpha^i, \quad (3.6)$$

$$F_{z\bar{z}} = \frac{1}{4}i\epsilon_{ij}\epsilon^{\alpha\beta}D_\alpha^iW_\beta^j, \quad (3.7)$$

Furthermore, W_α^i is constrained to satisfy

$$D_{(\beta}^{(j}W_{\alpha)}^i = 0, \quad \bar{D}_{(\dot{\beta}}^{(j}\bar{W}_{\dot{\alpha}}^i = 0, \quad (3.8)$$

$$\bar{D}_{\dot{\beta}}^{(j}W_{\alpha)}^i = 0, \quad D_{\beta}^{(j}\bar{W}_{\dot{\alpha}}^i = 0, \quad (3.9)$$

$$\epsilon^{\alpha\beta}D_\alpha^{[i}W_{\beta}^{j]} = -\epsilon^{\dot{\alpha}\dot{\beta}}\bar{D}_{\dot{\alpha}}^{[i}\bar{W}_{\dot{\beta}}^{j]}, \quad (3.10)$$

$$\epsilon^{\alpha\beta}D_\alpha^{(i}W_{\beta}^{j)} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{D}_{\dot{\alpha}}^{(i}\bar{W}_{\dot{\beta}}^{j)}. \quad (3.11)$$

Let us explore the consequences of (3.8)–(3.11) on the field content of W_α^i . The expansion of W_α^i in the anti-commuting coordinates has the general form

$$W_\alpha^i = \lambda_\alpha^i + \theta_j^\beta G_{\alpha\beta}^{ij} + \bar{\theta}_j^{\dot{\alpha}} H_{\alpha\dot{\alpha}}^{ij} + \mathcal{O}(\theta^2). \quad (3.12)$$

Conditions (3.8)–(3.11) will not impose any restriction on λ_α^i . First we consider the implications of the lowest component of superfield constraints (3.8)–(3.11). Condition (3.8) implies that

$$G_{\alpha\beta}^{ij} = i\epsilon^{ij}f_{\alpha\beta} + 2\epsilon^{ij}\epsilon_{\alpha\beta}D + i\epsilon_{\alpha\beta}\rho^{ij}, \quad (3.13)$$

where $f_{\alpha\beta} = f_{(\alpha\beta)}$ and $\rho^{ij} = \rho^{(ij)}$. Conditions (3.10) and (3.11) further imply the reality condition $D = D^\dagger$ and $\rho^{ij} = \bar{\rho}^{ij}$ where $\bar{\rho}_{ij} = (\rho^{ij})^\dagger$. Condition (3.9) yields

$$H_{\alpha\dot{\alpha}}^{ij} = i\epsilon^{ij}h_{\alpha\dot{\alpha}}. \quad (3.14)$$

Higher components of the superfield constraint (3.8)–(3.11) imply further conditions on the fields $f_{\alpha\beta}$, $h_{\alpha\dot{\alpha}}$, and D . One way to realize these conditions is to note that, from equations (3.4), (3.6), and (3.7),

$$\mathcal{F}_{ab} = F_{ab}| = -\frac{1}{8}\bar{\sigma}_a^{\dot{\alpha}\alpha}\bar{\sigma}_b^{\dot{\beta}\beta}\left(\epsilon_{\dot{\alpha}\dot{\beta}}f_{\alpha\beta} + \epsilon_{\alpha\beta}\bar{f}_{\dot{\alpha}\dot{\beta}}\right), \quad (3.15)$$

$$F_{a\bar{z}}| = \frac{1}{2}h_a, \quad (3.16)$$

$$F_{z\bar{z}}| = 2iD, \quad (3.17)$$

where $\bar{f}_{\dot{\alpha}\dot{\beta}} = (f_{\alpha\beta})^\dagger$ and $h_a = -\frac{1}{2}\sigma_a^{\alpha\dot{\alpha}}h_{\alpha\dot{\alpha}}$. The equations (3.7)–(3.11) are the general solution of the Bianchi identities subject to our constraints. It was shown in [1],

that three of these Bianchi identities imply that \mathcal{F}_{ab} is the field strength of a gauge field V_a

$$\mathcal{F}_{ab} = \partial_a V_b - \partial_b V_a. \quad (3.18)$$

and that

$$\partial_{[a} h_{b]}^R = -\frac{1}{2}(\partial_z + \partial_{\bar{z}})\mathcal{F}_{ab}, \quad (3.19)$$

$$\partial_{[a} h_{b]}^I = -\frac{1}{2}i(\partial_z - \partial_{\bar{z}})\mathcal{F}_{ab}, \quad (3.20)$$

$$\partial_a D = -\frac{1}{4}i \{ (\partial_z - \partial_{\bar{z}})h_a^R + i(\partial_z + \partial_{\bar{z}})h_a^I \}, \quad (3.21)$$

where $h_a = h_a^R + ih_a^I$.

This is as far as we can go using solely the constraints (3.3). So far we have the following component fields: an SU(2) doublet of spinors λ_α^i , a real gauge field V_a , a complex vector field h_a , a real scalar D and a real SU(2) triplet of scalars ρ^{ij} . We would like to impose further constraints to reduce the number of fields to an irreducible multiplet. First consider the fields D and ρ^{ij} ? We will find that D and ρ^{ij} play the role of auxiliary fields. Note that the usual $N = 2$ gauge multiplet has a triplet of auxiliary fields. Hence, if we want to derive the usual gauge multiplet we are led to set D to zero by promoting the constraint (3.10) to the stronger one

$$\epsilon^{\alpha\beta} D_\alpha^{[i} W_\beta^{j]} = 0, \quad \epsilon^{\dot{\alpha}\dot{\beta}} \bar{D}_{\dot{\alpha}}^{[i} \bar{W}_{\dot{\beta}}^{j]} = 0. \quad (3.22)$$

We will discuss this constraint, in the non-Abelian context, later. On the other hand, if we want to derive the vector-tensor multiplet, which has a single auxiliary field, we must eliminate ρ^{ij} . This can be achieved by promoting condition (3.11) to the stronger condition

$$\epsilon^{\alpha\beta} D_\alpha^{(i} W_\beta^{j)} = 0, \quad \epsilon^{\dot{\alpha}\dot{\beta}} \bar{D}_{\dot{\alpha}}^{(i} \bar{W}_{\dot{\beta}}^{j)} = 0. \quad (3.23)$$

Henceforth, we add condition (3.23) as a further constraint on the theory. Now let us proceed using conditions (3.8)–(3.10) as well as the stronger version of (3.11), constraint (3.23). Apart from eliminating ρ^{ij} , it can be shown that the consequences of imposing (3.23) are

$$\begin{aligned} \partial^a h_a^R &= i(\partial_z - \partial_{\bar{z}})D, \\ \partial^a h_a^I &= -(\partial_z + \partial_{\bar{z}})D, \\ \partial^b \mathcal{F}_{ab} &= -\frac{1}{4} \{ (\partial_z + \partial_{\bar{z}})h_a^R + i(\partial_z - \partial_{\bar{z}})h_a^I \}. \end{aligned} \quad (3.24)$$

We would like to render the real and imaginary components of h_a as field strengths. To do this, we must impose yet another, and final, constraint. We take a reality condition on the central charge

$$\partial_z W_\alpha^i = \partial_{\bar{z}} W_\alpha^i. \quad (3.25)$$

This means that superfield W_α^i , and hence all the curvature tensor components, depend on the two central charges z and \bar{z} only through the combination $z + \bar{z}$ with no dependence on $z - \bar{z}$. Substituting from constraint (3.25) into (3.20) and (3.24) immediately gives the following constraints on the real and imaginary components of h_a

$$\partial^a h_a^R = 0, \quad \partial_{[a} h_{b]}^I = 0. \quad (3.26)$$

They assert that h_a^R and h_a^I are field strengths of an anti-symmetric tensor and a scalar field respectively. That is

$$\begin{aligned} h_a^R &= \frac{1}{3} \epsilon_{abcd} H^{bcd} = \frac{1}{3} \epsilon_{abcd} \partial^b B^{cd}, \\ h_a^I &= 2 \partial_a \phi. \end{aligned} \quad (3.27)$$

In conclusion, we find that the component fields in W_α^i are exactly those of the vector-tensor multiplet, namely $(\lambda_\alpha^i, \phi, B_{ab}, V_a, D)$. These fields are actually functions of the central charge coordinates as well as the spacetime coordinates. However, by virtue of constraint (3.25), they depend on the central charge coordinates only via $z + \bar{z}$. Let us introduce a real central charge coordinate $s = z + \bar{z}$. We now show that the above conditions completely determine their dependence on s in terms of their lowest component in their central charge expansion. To see this, let us summarize the relevant conditions in (3.20), (3.21), and (3.24) derived on the bosonic fields. They are

$$\begin{aligned} \partial_s D &= -\frac{1}{2} \partial^a h_a^I, \\ \partial_s h_a^R &= -2 \partial^b \mathcal{F}_{ab}, \\ \partial_s h_a^I &= 2 \partial_a D, \\ \partial_s \mathcal{F}_{ab} &= -\partial_{[a} h_{b]}^R. \end{aligned} \quad (3.28)$$

Equations (3.28) constitute a system of first-order “differential equations” for the s dependence of the bosonic fields. The general solution is completely determined in terms of the “initial conditions”; that is, the values of the fields at $s = 0$, say, $D(x^m)$, $h_a(x^m)$, and $\mathcal{F}_{ab}(x^m)$. Thus in a Taylor series expansion in s each term in the series gets related to lower-order terms, so that for instance,

$$D(x^m, s) = D(x^m) - \frac{1}{2} \partial^a h_a^I(x^m) s - \frac{1}{2} \square D(x^m) s^2 + \cdots, \quad (3.29)$$

with similar formulas for h_a^R , h_a^I , and \mathcal{F}_{ab} . Furthermore, in [1] we showed that

$$\partial_s \lambda_\alpha^i = \sigma^a_{\alpha\dot{\alpha}} \partial_a \bar{\lambda}^{\dot{\alpha}i}, \quad (3.30)$$

where $\bar{\lambda}_{\dot{\alpha}i} = (\lambda_\alpha^i)^\dagger$. Equation (3.30) is the relation for the fermionic field λ_α^i corresponding to equations (3.28) for the bosonic fields. It fixes the expansion of λ_α^i in the central charge s , leaving only the lowest component $\lambda_\alpha^i(x^m)$ arbitrary.

We now compute the supersymmetry transformation of the different component fields. To do this we have to act on W_α^i with

$$\delta_\xi = \xi_i^\alpha Q_\alpha^i + \bar{\xi}_{\dot{\alpha}}^i \bar{Q}_i^{\dot{\alpha}}. \quad (3.31)$$

We find

$$\begin{aligned} \delta_\xi D &= -\frac{1}{2}i \left(\xi_i^\alpha \sigma_{\alpha\dot{\alpha}}^a \partial_a \bar{\lambda}^{\dot{\alpha}i} + \bar{\xi}_{\dot{\alpha}}^i \sigma_{\alpha\dot{\alpha}}^a \partial_a \lambda^{\alpha i} \right), \\ \delta_\xi \phi &= \frac{1}{2}i \left(\xi_{\alpha i} \lambda^{\alpha i} - \bar{\xi}_{\dot{\alpha} i} \bar{\lambda}^{\dot{\alpha} i} \right), \\ \delta_\xi B_{cd} &= \frac{1}{6}\epsilon_{abcd} \sigma^{ab}_{(\alpha\beta)} \xi_i^\alpha \lambda^{\beta j} + \frac{1}{6}\epsilon_{abcd} \bar{\sigma}^{ba}_{(\dot{\alpha}\dot{\beta})} \bar{\xi}_{\dot{\alpha}}^i \bar{\lambda}^{\dot{\beta} j}, \\ \delta_\xi V_a &= -\frac{1}{2}\xi_{\alpha i} \bar{\sigma}_a^{\dot{\alpha}\alpha} \bar{\lambda}_{\dot{\alpha}}^i + \frac{1}{2}\bar{\xi}_{\dot{\alpha} i} \bar{\sigma}_a^{\dot{\alpha}\alpha} \lambda_\alpha^i, \\ \delta_\xi \lambda_\alpha^i &= 2i\xi^{\beta i} \epsilon^{\dot{\alpha}\dot{\beta}} \sigma_{\alpha\dot{\alpha}}^a \sigma_{\beta\dot{\beta}}^b \partial_{[a} V_{b]} + 2\xi_\alpha^i D + \frac{i}{3}\bar{\xi}^{\dot{\alpha} i} \sigma_{\alpha\dot{\alpha}}^a \epsilon_{abcd} \partial^b B^{cd} - 2\sigma_{\alpha\dot{\alpha}}^a \bar{\xi}^{\dot{\alpha}} \partial_a \phi. \end{aligned} \quad (3.32)$$

The fields in these supersymmetry transformations are formally functions of both x^m and s , but (3.32) also express the variations of the lowest-order independent fields, which are functions of x^m only.

To get the central charge transformations we have to act with $\delta_\omega = \omega \partial_s$ on W_α^i . It is straightforward to get the following set of transformations

$$\begin{aligned} \delta_\omega D &= -\omega \square \phi, \\ \delta_\omega \phi &= \omega D, \\ \delta_\omega B^{cd} &= 3\omega \epsilon^{abcd} \partial_a V_b, \\ \delta_\omega V_a &= -\frac{1}{6}\omega \epsilon_{abcd} \partial^b B^{cd}, \\ \delta_\omega \lambda_\alpha^i &= \omega \sigma_{\alpha\dot{\alpha}}^a \partial_a \bar{\lambda}^{\dot{\alpha} i}. \end{aligned} \quad (3.33)$$

Expressions (3.32) and (3.33) reproduce the supersymmetry and central charge transformations of the vector-tensor multiplet given in the component field calculations of [2, 3, 6].

We now give the superfield action for the pure vector-tensor multiplet. In [1] it was found that the appropriate action is

$$\begin{aligned} S &= \int d^4x \left\{ -\frac{1}{192} D_i^\alpha D_{\alpha j} W^{\beta i} W_\beta^j + \frac{1}{192} \bar{D}_{\dot{\alpha} i} \bar{D}_{\dot{\alpha} j} W^{\beta i} W_\beta^j + \text{h.c.} \right\} \\ &= \int d^4x \left\{ -\frac{1}{4} \mathcal{F}_{ab} \mathcal{F}^{ab} - \frac{1}{2} \partial_a \phi \partial^a \phi - \frac{1}{12} \partial_a B_{bc} \partial^a B^{bc} + \frac{1}{2} D^2 - \frac{1}{4} i \lambda^{\alpha i} \sigma_{\alpha\dot{\alpha}}^a \partial_a \bar{\lambda}_{\dot{\alpha}}^i \right\}. \end{aligned} \quad (3.34)$$

This represents a central charge generalization of the type of superactions considered in [11]. This completes our brief discussion of the gauge superfield formulation of the vector-tensor multiplet.

4. INTEGRABILITY AND THE SUPERFIELD W

The results of the previous section are sufficient to describe the vector- tensor multiplet. However, there is an, as yet, undiscussed property of this theory which gives considerable insight into the nature of this multiplet. The constraints on superfield W_α^i are given in (3.8)-(3.10), (3.23) and (3.25). These constraints are all first order differential conditions. Further differentiation of these constraints leads to a number of integrability conditions. These are sufficient to allow one to completely solve for W_α^i in terms of a real scalar superfield. This scalar superfield is subject to predominantly second order constraints. Specifically, we find that

$$W_\alpha^i = iD_\alpha^i W, \quad (4.1)$$

where scalar superfield W is real, that is $W^\dagger = W$. The form of (4.1) follows immediately from the W_α^i constraint (3.8) and the fact that

$$D_{(\alpha}^{(i} D_{\beta)}^{j)} = 0 \quad (4.2)$$

The superfield W is subject to a set of constraints that can be obtained by substituting (4.1) into (3.9), (3.10), (3.23), and (3.25). Substituting expression (4.1) into W_α^i constraints (3.9) and (3.23) implies that W must satisfy the conditions

$$D_\alpha^{(i} \bar{D}_{\dot{\beta}}^{j)} W = 0, \quad \bar{D}_{\dot{\alpha}}^{(i} D_{\beta}^{j)} W = 0 \quad (4.3)$$

and

$$\epsilon^{\alpha\beta} D_\alpha^{(i} D_\beta^{j)} W = 0, \quad \epsilon^{\dot{\alpha}\dot{\beta}} \bar{D}_{\dot{\alpha}}^{(i} \bar{D}_{\dot{\beta}}^{j)} W = 0 \quad (4.4)$$

respectively. Finally, using the fact that

$$D_{[\alpha}^{[i} D_{\beta]}^{j]} = -i\epsilon^{ij}\epsilon_{\alpha\beta}\partial_z \quad (4.5)$$

we find from W_α^i constraint (3.10) that

$$\partial_z W = \partial_{\bar{z}} W. \quad (4.6)$$

The appearance of W in this gauge formulation is, at first sight, somewhat surprising since all the curvature tensor components are given in terms of W_α^i and its derivatives. However, as we will show in a forthcoming paper [12], there exists a one-parameter class of constraints, all of which describe the vector-tensor multiplet. The set of constraints that we use in this paper is a single instance of this class of constraints. In all but this particular instance, the scalar superfield W appears explicitly and naturally as a component of the curvature tensor.

Note that since W_α^i is a gauge-invariant object, so is W . Since W_α^i and, hence, all other physically relevant superfields can be obtained from W by differentiation,

it is clear that W plays a fundamental role in the description of the vector tensor-multiplet. In particular, W should contain all the component fields of the vector-tensor multiplet. Indeed it does, but in a somewhat non-trivial way. To see this, let us postulate the following expansion of W in the anti-commuting coordinates

$$W = 2\phi - i\theta_i^\alpha \lambda_\alpha^i + i\bar{\theta}_i^{\dot{\alpha}} \bar{\lambda}_{\dot{\alpha}}^i + \theta_i^\alpha \bar{\theta}^{\dot{\alpha}i} h_{\alpha\dot{\alpha}}^R + \frac{1}{2}\theta_i^\alpha \theta^{\beta i} f_{\alpha\beta} - \frac{1}{2}\bar{\theta}_i^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}i} \bar{f}_{\dot{\alpha}\dot{\beta}} + \mathcal{O}(\theta^3). \quad (4.7)$$

We remind the reader that the component fields satisfy (3.15), (3.18) and (3.27). Substituting this into relation (4.1), we find that it correctly reproduces the component expansion (3.12)-(3.14) of W_α^i as long as one can identify the derivative term $\partial_s \phi$ with the auxiliary field D . This identification can be justified using the central charge conditions (3.28). Specifically, combining the definition of ϕ in (3.27) with the equations (3.28), we find that

$$\phi(x^m, s) = \phi(x^m) + D(x^m)s - \frac{1}{2}\square\phi(x^m)s^2 + \dots \quad (4.8)$$

and therefore

$$D = \partial_s \phi|_{s=0} \quad (4.9)$$

Conditions (3.28) further imply that all other coefficients in the s expansion of W are determined in terms of $\lambda_\alpha^i, \phi, B_{ab}, V_a$ and D . We conclude that (4.7) is indeed the correct component expansion of scalar superfield W and that it contains all of the component fields $\lambda_\alpha^i, \phi, B_{ab}, V_a$ and D . It is interesting to note that the component fields $\lambda_\alpha^i, \phi, B_{ab}$ and V_a all occur at zeroth order in the central charge, whereas auxiliary field D appears at first order in the Taylor expansion of ϕ in the central charge s . This is similar to the appearance of the auxiliary fields ϕ_i in the Fayet-Sohnius multiplet. In that superfield, the lowest order piece gives the dynamically propagating fields A_i , while the first order term in the s -expansion gives the auxiliary fields. In the above discussion, we derived the component expansion of W making use of our previous results for W_α^i . It is important to emphasize, however, that this is not necessary. The conditions that superfield W be real and satisfy constraints (4.3), (4.4) and (4.6) are sufficient to completely determine the component structure. Furthermore, the supersymmetry transformations (3.32) and the central charge transformations (3.33) can be determined directly by acting on W with (3.31) and $\delta_\omega = \omega \partial_s$.

We close by emphasizing that the vector-tensor multiplet is formulated in this section as a gauge multiplet, where the component gauge field V_a is introduced directly in the one-form connection and the appearance of the antisymmetric tensor component field B_{ab} is a consequence of applying the appropriate constraints. Note, however, that the $N = 2$ superfield W bears a close resemblance to the real linear multiplet superfield of $N = 1$ supersymmetry. For $N = 1$ theories, a geometric superspace formulation of the linear multiplet in terms of a two-form multiplet was

constructed in [13]. This suggests that the $N = 2$ vector-tensor multiplet has a similar formulation in terms of an $N = 2$ two-form multiplet as well. This will be the subject of Section 5.

5. VECTOR-TENSOR MULTIPLT: TWO-FORM FORMULATION

One begins by introducing a real two-form superfield in superspace with central charge.

$$B = \frac{1}{2}e^B e^A B_{AB} \quad (5.1)$$

where e^A denotes the frame of flat superspace. The invariant field strength is the three-form

$$H = dB \quad (5.2)$$

Its component superfields, defined by

$$H = \frac{1}{3!}e^C e^B e^A H_{ABC} \quad (5.3)$$

are then subject to a set of constraints. The first of these stipulates that any completely spinorial component of H must vanish. That is

$$H_{\underline{\alpha}\beta\gamma}^{ijk} = 0. \quad (5.4)$$

The second constraint is given by

$$H_{\alpha\beta a}^{ij} = 0, \quad H_{\dot{\alpha}\dot{\beta} a}^{ij} = 0. \quad (5.5)$$

The third, and final, constraint is that

$$H_{\alpha\beta a}^{ij} = H_{\beta\alpha a}^{ji} = 2i\epsilon^{ij}\sigma_{a\alpha\dot{\beta}}L \quad (5.6)$$

where L is a real superfield. We now solve the Bianchi identities, $dH = 0$, subject to these constraints. We find that all component superfields of H vanish except for the following

$$H_{\alpha\beta\bar{z}}^{ij} = -4i\epsilon^{ij}\epsilon_{\alpha\beta}L, \quad H_{\dot{\alpha}\dot{\beta}z}^{ij} = -4i\epsilon^{ij}\epsilon_{\dot{\alpha}\dot{\beta}}L, \quad (5.7)$$

$$H_{\gamma\bar{z}z}^k = 2D_{\gamma}^k L, \quad H_{\dot{\gamma}z\bar{z}}^k = 2D_{\dot{\gamma}}^k L, \quad (5.8)$$

$$H_{\alpha\bar{z}a}^i = -2\epsilon_{\alpha\beta}\bar{\sigma}_a^{\dot{\gamma}\beta}\bar{D}_{\dot{\gamma}}^i L, \quad H_{\dot{\alpha}za}^i = 2\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}_a^{\beta\dot{\gamma}}D_{\dot{\gamma}}^i L, \quad (5.9)$$

$$H_{\gamma ab}^k = -2\sigma_{ab\gamma}^{\alpha}D_{\alpha}^k L, \quad H_{\dot{\gamma}ab}^k = 2\bar{\sigma}_{ab}^{\dot{\alpha}}\bar{D}_{\dot{\alpha}}^k L, \quad (5.10)$$

$$H_{zab} = -\frac{i}{2}\epsilon_{ij}\epsilon^{\alpha\gamma}\sigma_{ab\gamma}^{\beta}D_{\alpha}^i D_{\beta}^j L, \quad H_{\bar{z}ab} = \frac{i}{2}\epsilon_{ij}\epsilon^{\dot{\alpha}\dot{\gamma}}\bar{\sigma}_{ab}^{\dot{\beta}}\bar{D}_{\dot{\alpha}}^i \bar{D}_{\dot{\beta}}^j L, \quad (5.11)$$

$$H_{a\bar{z}\bar{z}} = -\frac{i}{4}\epsilon_{ij}\bar{\sigma}_a^{\dot{\alpha}\alpha}[D_{\alpha}^i, \bar{D}_{\dot{\alpha}}^j]L, \quad H_{abc} = -\frac{1}{8}\epsilon_{abcd}\bar{\sigma}^{d\dot{\beta}\alpha}\epsilon_{ij}[D_{\alpha}^i, \bar{D}_{\dot{\beta}}^j]L. \quad (5.12)$$

Furthermore, superfield L is required to satisfy

$$D_\alpha^{(i} \bar{D}^{j)} L = 0 \quad (5.13)$$

as well as

$$\epsilon^{\alpha\beta} D_\alpha^{(i} D_\beta^{j)} L = 0, \quad \epsilon^{\dot{\alpha}\dot{\beta}} \bar{D}_{\dot{\alpha}}^{(i} \bar{D}_{\dot{\beta}}^{j)} L = 0 \quad (5.14)$$

It is important to note that these conditions on L are the same as constraints (4.3) and (4.4) on superfield W . If we impose one additional constraint on L , namely that

$$\partial_z L = \partial_{\bar{z}} L \quad (5.15)$$

then the constraints on L are the exactly same as those on W . As discussed at the end of the previous section, the conditions that superfield L be real and satisfy constraints (4.3), (4.4) and (4.6) are sufficient to completely determine its component field structure. Since the constraints are identical to those on W , it follows that superfield L describes the vector-tensor multiplet with component fields $\lambda_\alpha^i, \phi, B_{ab}, V_a$ and D . Furthermore, the supersymmetry transformations (3.32) and the central charge transformations (3.33) can be determined directly by acting on L with (3.31) and $\delta_\omega = \omega \partial_s$. That is, we can identify

$$L = W \quad (5.16)$$

We conclude that the vector-tensor multiplet can be equivalently described by introducing the two-form component field B_{ab} directly through a two-form superfield and deriving the gauge component field V_a by applying the appropriate constraints.

6. NON-ABELIAN SUPERAUGE MULTIPLYTS AND CHERN-SIMONS FORMS

Having presented both a supergauge field formulation and a super three-form formulation of the vector-tensor multiplet, we now return to the generic supergauge theory of Section 3. We modify that theory so as to describes usual $N = 2$ supergauge multiplets associated with a non-Abelian gauge group. This modification is very straightforward. We begin by introducing a Lie algebra valued Hermitian connection $\mathcal{A} = dz^M \mathcal{A}_M = e^A \mathcal{A}_A$. The hermiticity of the connection then implies

$$\mathcal{A} = dx^a \mathcal{A}_a + d\theta_i^\alpha \mathcal{A}_\alpha^i + d\bar{\theta}_{\dot{\alpha}}^i \bar{\mathcal{A}}_{\dot{\alpha}}^i + dz \mathcal{A}_z + d\bar{z} \mathcal{A}_{\bar{z}}, \quad (6.1)$$

where \mathcal{A}_a is real, $\bar{\mathcal{A}}_{\dot{\alpha}}^i = (\mathcal{A}^{\alpha i})^\dagger$, and $\mathcal{A}_{\bar{z}} = \mathcal{A}_z^\dagger$. The curvature two-form is defined as

$$\mathcal{F} = d\mathcal{A} + \mathcal{A}\mathcal{A} = \frac{1}{2} e^B e^A \mathcal{F}_{AB}. \quad (6.2)$$

The curvature tensor \mathcal{F} is subject to the Bianchi identities $\mathcal{D}\mathcal{F} = 0$. As in the Abelian case, we will adopt a set of constraints which set the pure spinorial part of

the curvature tensor to zero. That is

$$\mathcal{F}_{\alpha\beta}^{ij} = \mathcal{F}_{\alpha\dot{\beta}}^{ij} = \mathcal{F}_{\dot{\alpha}\dot{\beta}}^{ij} = 0. \quad (6.3)$$

Before imposing further conditions we would like to explore the consequences of (6.3). To do so we must solve the Bianchi identities subject to these constraints. The result is that all the components of the curvature tensor \mathcal{F}_{AB} are determined in terms of a single superfield $\mathcal{F}_{\alpha\bar{z}}^i$ and its hermitian conjugate $\mathcal{F}_{\dot{\alpha}iz} = (\mathcal{F}_{\alpha\bar{z}}^i)^\dagger$. Henceforth, we denote these superfields by \mathcal{W}_α^i and $\bar{\mathcal{W}}_{\dot{\alpha}i}$. In particular $\mathcal{F}_{\alpha z}^i = 0$ and

$$\mathcal{F}_{ab} = -\frac{1}{16}i\epsilon_{ij}\bar{\sigma}_a^{\dot{\alpha}\alpha}\bar{\sigma}_b^{\dot{\beta}\beta}\left(\epsilon_{\dot{\alpha}\dot{\beta}}\mathcal{D}_\beta^j\mathcal{W}_\alpha^i + \epsilon_{\alpha\beta}\bar{\mathcal{D}}_{\dot{\beta}}^j\bar{\mathcal{W}}_{\dot{\alpha}}^i\right). \quad (6.4)$$

$$\mathcal{F}_{\dot{\alpha}a}^i = -\frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}_a^{\dot{\beta}\alpha}\mathcal{W}_\alpha^i, \quad (6.5)$$

$$\mathcal{F}_{a\bar{z}} = -\frac{1}{8}i\epsilon_{ij}\bar{\sigma}_a^{\dot{\beta}\alpha}\bar{\mathcal{D}}_{\dot{\beta}}^j\mathcal{W}_\alpha^i, \quad (6.6)$$

$$\mathcal{F}_{z\bar{z}} = \frac{1}{4}i\epsilon_{ij}\epsilon^{\alpha\beta}\mathcal{D}_\alpha^i\mathcal{W}_\beta^j. \quad (6.7)$$

Furthermore, \mathcal{W}_α^i is constrained to satisfy

$$\mathcal{D}_{(\beta}^{(j}\mathcal{W}_{\alpha)}^i = 0, \quad \bar{\mathcal{D}}_{(\dot{\beta}}^{(j}\bar{\mathcal{W}}_{\dot{\alpha}}^i = 0, \quad (6.8)$$

$$\bar{\mathcal{D}}_{\dot{\beta}}^{(j}\mathcal{W}_{\alpha)}^i = 0, \quad \mathcal{D}_{\beta}^{(j}\bar{\mathcal{W}}_{\dot{\alpha}}^i = 0, \quad (6.9)$$

$$\epsilon^{\alpha\beta}\mathcal{D}_\alpha^{[i}\mathcal{W}_{\beta}^{j]} = -\epsilon^{\dot{\alpha}\dot{\beta}}\bar{\mathcal{D}}_{\dot{\alpha}}^{[i}\bar{\mathcal{W}}_{\dot{\beta}}^{j]}, \quad (6.10)$$

$$\epsilon^{\alpha\beta}\mathcal{D}_\alpha^{(i}\mathcal{W}_{\beta}^{j)} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\mathcal{D}}_{\dot{\alpha}}^{(i}\bar{\mathcal{W}}_{\dot{\beta}}^{j)}. \quad (6.11)$$

Now, as discussed in Section 3, it is necessary to impose additional constraints to further reduce the number of fields to an irreducible supermultiplet. In order to obtain the usual gauge multiplet in the Abelian case, one introduces constraint (3.22). The non-Abelian generalization of this is straightforward and is obtained by promoting constraint (6.10) to the stronger one

$$\epsilon^{\alpha\beta}\mathcal{D}_\alpha^{[i}\mathcal{W}_{\beta}^{j]} = 0, \quad \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\mathcal{D}}_{\dot{\alpha}}^{[i}\bar{\mathcal{W}}_{\dot{\beta}}^{j]} = 0. \quad (6.12)$$

Note that this additional constraint is equivalent to setting

$$\mathcal{F}_{z\bar{z}} = 0. \quad (6.13)$$

Furthermore, it is necessary to add a reality condition on the central charge given by

$$\mathcal{D}_z\mathcal{W}_\alpha^i = \mathcal{D}_{\bar{z}}\mathcal{W}_\alpha^i, \quad \mathcal{D}_z\bar{\mathcal{W}}_{\dot{\alpha}}^i = \mathcal{D}_{\bar{z}}\bar{\mathcal{W}}_{\dot{\alpha}}^i \quad (6.14)$$

Constraints (6.3), (6.12) (or, equivalently, (6.13)) and (6.14) are sufficient to completely describe the non-Abelian supergauge multiplet. Now, as in the case of the vector-tensor multiplet, these constraints are such as to allow us to completely solve

for \mathcal{W}_α^i and $\bar{\mathcal{W}}_\alpha^i$ in terms of two complex scalar superfields \mathcal{W} and $\bar{\mathcal{W}}$. Specifically, we find that

$$\mathcal{W}_\alpha^i = i\mathcal{D}_\alpha^i \mathcal{W}, \quad \bar{\mathcal{W}}_\alpha^i = -i\bar{\mathcal{D}}_\alpha^i \bar{\mathcal{W}}, \quad (6.15)$$

where scalar superfield \mathcal{W} is complex and $\bar{\mathcal{W}} = \mathcal{W}^\dagger$. The form of (6.15) follows immediately from the constraint (6.8) and the fact that

$$\mathcal{D}_{(\alpha}^{(i} \mathcal{D}_{\beta)}^{j)} = 0. \quad (6.16)$$

The superfields \mathcal{W} and $\bar{\mathcal{W}}$ are subject to a set of constraints that can be obtained by substituting (6.15) into (6.9), (6.11), (6.12) and (6.14). Substituting expression (6.15) into constraints (6.9) and (6.11) implies that \mathcal{W} and $\bar{\mathcal{W}}$ must satisfy the conditions

$$\bar{\mathcal{D}}_\alpha^{(i} \mathcal{D}_\beta^{j)} \mathcal{W} = 0, \quad \mathcal{D}_\alpha^{(i} \bar{\mathcal{D}}_\beta^{j)} \bar{\mathcal{W}} = 0 \quad (6.17)$$

and

$$\epsilon^{\alpha\beta} \mathcal{D}_\alpha^{(i} \mathcal{D}_\beta^{j)} \mathcal{W} = \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\mathcal{D}}_{\dot{\alpha}}^{(i} \bar{\mathcal{D}}_{\dot{\beta}}^{j)} \bar{\mathcal{W}} \quad (6.18)$$

respectively. Using the fact that

$$\mathcal{D}_{[\alpha}^{[i} \mathcal{D}_{\beta]}^{j]} = -i\epsilon^{ij} \epsilon_{\alpha\beta} \mathcal{D}_z, \quad (6.19)$$

we find from constraint (6.12) that

$$\mathcal{D}_z \mathcal{W} = 0, \quad \mathcal{D}_{\bar{z}} \bar{\mathcal{W}} = 0. \quad (6.20)$$

Combining this result with constraint (6.14) we have, finally, that

$$\mathcal{D}_z \mathcal{W} = \mathcal{D}_{\bar{z}} \mathcal{W} = 0, \quad (6.21)$$

$$\mathcal{D}_z \bar{\mathcal{W}} = \mathcal{D}_{\bar{z}} \bar{\mathcal{W}} = 0. \quad (6.22)$$

As will be discussed elsewhere, it is possible, using the redundancy in the solution of (6.15), to choose \mathcal{W} and $\bar{\mathcal{W}}$ to satisfy $\bar{\mathcal{D}}_\alpha^i \mathcal{W} = \mathcal{D}_\alpha^i \bar{\mathcal{W}} = 0$. We do this henceforth.

The Chern-Simons form associated with this non-Abelian supergauge theory is defined to be

$$\mathcal{Q} = \text{tr} (\mathcal{A}\mathcal{F} - \frac{1}{3}\mathcal{A}\mathcal{A}\mathcal{A}) \quad (6.23)$$

The Chern-Simons term can be coupled to the super two-form B discussed in the previous section by modifying the three-form superfield strength H to become

$$\mathcal{H} = dB + \kappa \mathcal{Q} \quad (6.24)$$

where κ is an arbitrary real coupling parameter. Its component superfields, defined by

$$\mathcal{H} = \frac{1}{3!} e^C e^B e^A \mathcal{H}_{ABC} \quad (6.25)$$

are then subject to the constraints

$$\mathcal{H}_{\underline{\alpha\beta\gamma}}^{ijk} = 0. \quad (6.26)$$

$$\mathcal{H}_{\alpha\beta a}^{ij} = 0, \quad \mathcal{H}_{\dot{\alpha}\dot{\beta} a}^{ij} = 0. \quad (6.27)$$

$$\mathcal{H}_{\alpha\dot{\beta} a}^{ij} = \mathcal{H}_{\dot{\beta}\alpha a}^{ji} = 2i\epsilon^{ij}\sigma_{a\alpha\dot{\beta}}\mathcal{L} \quad (6.28)$$

where \mathcal{L} is a real superfield. The addition of the Chern-Simons form modifies the Bianchi identities to

$$d\mathcal{H} = \kappa \operatorname{tr} \mathcal{F}\mathcal{F} \quad (6.29)$$

We now solve these Bianchi identities subject to the above constraints. We find that

$$\mathcal{H}_{\alpha\beta\bar{z}}^{ij} = -4i\epsilon^{ij}\epsilon_{\alpha\beta}\mathcal{L}, \quad \mathcal{H}_{\dot{\alpha}\dot{\beta}z}^{ij} = -4i\epsilon^{ij}\epsilon_{\dot{\alpha}\dot{\beta}}\mathcal{L}. \quad (6.30)$$

$$\mathcal{H}_{\gamma\bar{z}z}^k = 2D_{\gamma}^k\mathcal{L}, \quad \mathcal{H}_{\dot{\gamma}z\bar{z}}^k = 2D_{\dot{\gamma}}^k\mathcal{L}. \quad (6.31)$$

$$\mathcal{H}_{\alpha\bar{z}a}^i = -2\epsilon_{\alpha\beta}\bar{\sigma}_a^{\dot{\gamma}\beta}\bar{D}_{\dot{\gamma}}^i\mathcal{L}, \quad \mathcal{H}_{\dot{\alpha}za}^i = 2\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}_a^{\beta\dot{\gamma}}D_{\dot{\gamma}}^i\mathcal{L}. \quad (6.32)$$

$$\mathcal{H}_{\gamma ab}^k = -2\sigma_{ab\gamma}^{\alpha}D_{\alpha}^k\mathcal{L}, \quad \mathcal{H}_{\dot{\gamma}ab}^k = 2\bar{\sigma}_{ab}^{\dot{\alpha}\gamma}\bar{D}_{\dot{\alpha}}^k\mathcal{L}, \quad (6.33)$$

$$\mathcal{H}_{zab} = -\frac{i}{2}\epsilon_{ij}\epsilon^{\alpha\gamma}\sigma_{ab\gamma}^{\beta}D_{\alpha}^iD_{\beta}^j\mathcal{L} + ik\epsilon_{ij}\epsilon^{\beta\dot{\gamma}}\bar{\sigma}_{ab}^{\dot{\alpha}\gamma}\operatorname{tr}(\bar{\mathcal{W}}_{\dot{\alpha}}^i\bar{\mathcal{W}}_{\beta}^j), \quad (6.34)$$

$$\mathcal{H}_{\bar{z}ab} = \frac{i}{2}\epsilon_{ij}\epsilon^{\dot{\alpha}\dot{\gamma}}\bar{\sigma}_{ab}^{\dot{\beta}\gamma}\bar{D}_{\dot{\alpha}}^i\bar{D}_{\dot{\beta}}^j\mathcal{L} - ik\epsilon_{ij}\epsilon^{\beta\gamma}\sigma_{ab\gamma}^{\alpha}\operatorname{tr}(\mathcal{W}_{\alpha}^i\mathcal{W}_{\beta}^j), \quad (6.35)$$

$$\mathcal{H}_{a\bar{z}z} = -\frac{i}{4}\epsilon_{ij}\bar{\sigma}_a^{\dot{\alpha}\alpha}[D_{\alpha}^i, \bar{D}_{\dot{\alpha}}^j]\mathcal{L} + 2ik\epsilon_{ij}\bar{\sigma}_a^{\alpha\dot{\alpha}}\operatorname{tr}(\mathcal{W}_{\alpha}^i\bar{\mathcal{W}}_{\dot{\alpha}}^j), \quad (6.36)$$

$$\mathcal{H}_{abc} = -\frac{1}{8}\epsilon_{abcd}\bar{\sigma}^{d\dot{\beta}\alpha}\epsilon_{ij}([D_{\alpha}^i, \bar{D}_{\dot{\beta}}^j]\mathcal{L} + 8\kappa\operatorname{tr}(\mathcal{W}_{\alpha}^i\bar{\mathcal{W}}_{\dot{\beta}}^j)). \quad (6.37)$$

Furthermore, superfield \mathcal{L} is constrained to satisfy

$$D_{\alpha}^{(i}\bar{D}_{\dot{\beta}}^{j)}\mathcal{L} = -2\kappa\operatorname{tr}(\mathcal{W}_{\alpha}^{(i}\bar{\mathcal{W}}_{\dot{\beta}}^{j)}) \quad (6.38)$$

as well as

$$\epsilon^{\alpha\beta}D_{\alpha}^{(i}D_{\beta}^{j)}\mathcal{L} = 4\kappa\operatorname{tr}(\bar{\mathcal{W}}_{\dot{\alpha}}^i\bar{\mathcal{W}}_{\dot{\beta}}^{j\dot{\alpha}}), \quad \epsilon^{\dot{\alpha}\dot{\beta}}\bar{D}_{\dot{\alpha}}^{(i}\bar{D}_{\dot{\beta}}^{j)}\mathcal{L} = 4\kappa\operatorname{tr}(\mathcal{W}_{\alpha}^i\mathcal{W}_{\alpha}^{j\dot{\alpha}}) \quad (6.39)$$

It follows that the addition of the Chern-Simons terms modifies both the solutions of the Bianchi identities and the constraints on \mathcal{L} . In particular, the constraints now include terms proportional to $\mathcal{W}_{\alpha}^{(i}\bar{\mathcal{W}}_{\dot{\beta}}^{j)}$ and $\mathcal{W}^{i\alpha}\mathcal{W}_{\alpha}^j$ and its conjugate. However, these constraints can be rewritten to be more closely analogous with (5.13) and

(5.14). Using (6.15) and the fact that $\bar{\mathcal{D}}_\alpha^i \mathcal{W} = \mathcal{D}_\alpha^i \bar{\mathcal{W}} = 0$, it is straightforward to show that (5.13) and (5.14) can be written in the form

$$D_\alpha^{(i} \bar{D}_{\dot{\beta}}^{j)} \Sigma = 0 \quad (6.40)$$

and

$$\epsilon^{\alpha\beta} D_\alpha^{(i} D_\beta^{j)} \Sigma + \epsilon^{\dot{\alpha}\dot{\beta}} \bar{D}_{\dot{\alpha}}^{(i} \bar{D}_{\dot{\beta}}^{j)} \Sigma = 0 \quad (6.41)$$

respectively, where

$$\Sigma = \mathcal{L} - 2\kappa \operatorname{tr} (\mathcal{W} + \bar{\mathcal{W}})^2 \quad (6.42)$$

As was discussed in [1], and used to construct the superfield action for the pure vector-tensor action (3.34), the generic form for a supersymmetric action in central charge superspace is necessarily of the form

$$S = \int d^4x \{ D_i^\alpha D_{\alpha j} - \bar{D}_{\dot{\alpha} i} \bar{D}_{\dot{j}}^{\dot{\alpha}} \} M^{ij} \quad (6.43)$$

where M^{ij} must be real, symmetric in the indices i and j and satisfy

$$D_{\underline{\alpha}}^{(i} M^{jk)} = 0 \quad (6.44)$$

For example, for the pure vector tensor multiplet discussed in Section 3, we chose $M^{ij} = -W^{\beta(i} W_{\beta}^{j)} + \bar{W}_{\dot{\beta}}^{(i} \bar{W}^{\dot{j})\dot{\beta}}$. In the case of the Chern-Simons modified theory, the appropriate action is given by (6.43) where

$$M^{ij} = D^{\beta(i} \Sigma D_{\beta}^{j)} \Sigma + \Sigma D^{\beta(i} D_{\beta}^{j)} \Sigma - \bar{D}_{\dot{\beta}}^{(i} \Sigma \bar{D}^{\dot{j})\dot{\beta}} \Sigma \quad (6.45)$$

Note that this expression satisfies the above criteria. It is straightforward to expand this action into component fields. We will do this, and explore other issues involving theories of this type, in future publications.

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As this paper was being written, two related publications appeared. The first of these [14] discussed the two-form formulation of the vector-tensor multiplet and its coupling to Chern-Simons terms. The second paper [15] presented related topics within the context of harmonic superspace. We would like to thank Daniel Waldram for many important conversations.

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